Topological Methods in Nonlinear Analysis
Journal of the Juliusz Schauder Center
Volume 2, 1993, 293–315

ASYMPTOTIC BEHAVIOUR OF SOLUTIONS OF SEMILINEAR EQUATIONS WITH SUBCRITICAL NONLINEARITY

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Dedicated to the memory of Juliusz Schauder

1. Introduction

Let Ω be a bounded domain of $\mathbb{R}^n$ $(n \geq 2)$, with $C^{2,\sigma}$-boundary $\partial\Omega$ $(\sigma > 0)$. Consider the following problem:

$$P(\beta) \qquad \begin{cases} -\Delta u = \beta f(u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where f satisfies the following assumption:

(A) f is a positive, C^1-function on $\mathbb{R}$.

By the implicit function theorem, there exist an interval $I = (0, \widetilde{\beta})$ and a neighbourhood V of 0 in the space $C^{2,\sigma}(\overline{\Omega})$ such that, for any $\beta \in I$, there exists a unique solution $\widehat{u}(\cdot, \beta) \in V$ of the problem $P(\beta)$. Furthermore, for $\beta \in I$, $-\Delta - \beta f'(\widehat{u}(\cdot, \beta))$ has a positive first eigenvalue. (For the proof of the above fact, see Appendix.)

Suppose, in addition, that

(B) There are constants $C > 0$ and $1 < p < \widetilde{n}$ such that $f(u) < C(1 + u^p)$ for $u > 0$, where $\widetilde{n} = (n+2)/(n-2)$ if $n > 2$; $\widetilde{n} = +\infty$ if $n = 2$, and