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THE DEGREE OF PROPER C^2 FREDHOLM MAPPINGS: COVARIANT THEORY

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Dedicated to Jean Leray

1. Introduction

In a recent article [10] we introduced a degree for proper C^2 Fredholm mappings with index 0 in general Banach spaces. As subsequently shown in [11], where the case of Banach manifolds is also treated, this degree essentially coincides with the degree of Elworthy and Tromba [8] on completely orientable manifolds. Nevertheless, the conceptually much simpler approach taken in [10] not only facilitates the use of the degree in concrete applications, but also it completely clarifies its behavior under homotopy. Indeed, while it was already known that homotopy need not preserve the sign of the degree, it remained impossible to predict whether a sign change should occur until the new concept of parity was made a crucial part of the definition.

On the other hand, there is a fairly large body of literature devoted to the calculation of the degrees of Brouwer or of Leray-Schauder in the hypothesis that the mapping of interest is covariant under the action of a compact Lie group G. Naturally, the motivation for such studies can be found in Borsuk's theorem, dealing with the simplest case when $G = \mathbb{Z}_2$ acts through $\{I, -I\}$. Elegant theories about covariant properties of fixed point indices in topological spaces or

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