Topological Methods in Nonlinear Analysis
Journal of the Juliusz Schauder Center
Volume 3, 1994, 307–323

THE DIRICHLET PROBLEM FOR THE PRESCRIBED CURVATURE QUOTIENT EQUATIONS

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Dedicated to Jean Leray

1. Introduction

In this paper we consider the classical Dirichlet problem for equations of prescribed curvature of the form

$$F[u] = f(\kappa) = \Psi(x, u) \tag{1.1}$$

in domains Ω in Euclidean n-space, $\mathbb{R}^n$, where $\kappa = (\kappa_1, \dots, \kappa_n)$ denotes the principal curvatures of the graph of u over Ω, Ψ is a prescribed positive function on $\Omega \times \mathbb{R}$ and f is a symmetric function of the form

$$f(\kappa) = S_{k,l} = \frac{S_k}{S_l}, \tag{1.2}$$

where $0 \leq l < k \leq n$ and S_k denotes the k-th order elementary symmetric function,

$$S_k = \sum \kappa_{i_1} \kappa_{i_2} \dots \kappa_{i_k}, \tag{1.3}$$

the sum being taken over all increasing k-tuples $i_1, i_2, \dots, i_k \subset \{1, \dots, n\}$. Taking $S_0 = 1$, we may write $S_{k,0} = S_k$. The mean, scalar, Gauss and harmonic

[1]Research partially supported by Australian Research Council Grant.