Topological Methods in Nonlinear Analysis
Journal of the Juliusz Schauder Center
Volume 3, 1994, 235–256

A PRIORI ESTIMATES FOR THE GRADIENT OF THE SOLUTION TO THE SYSTEM OF VISCOELASTICITY IN SEVERAL DIMENSIONS

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Dedicated to Jean Leray

1. Introduction

The aim of this note is to study a priori estimates for the gradient of strong solutions u to the system of viscoelasticity

$$u_{tt} = \operatorname{div}(\sigma(\nabla u) + \nabla u_t), \qquad u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \tag{1}$$

on a bounded domain $\Omega \subset \mathbb{R}^n$ with smooth boundary; $u : \Omega \to \mathbb{R}^n$, $n > 1$. The main technical assumption is Lipschitz continuity of the nonlinearity and that σ is close to a linear mapping for large arguments. We consider data such that ∇u_0 is in the space of functions of bounded mean oscillation (BMO) or u_0, u_1 are spherically symmetric and ∇u_0 is essentially bounded. For the definition of BMO we refer the reader to Section 2 or to the original paper [13].

One can hope for such estimates because of a hidden parabolic structure of (1). Its parabolic structure may be made transparent after a diagonalization procedure (cf. [14]–[16]): the introduction of new variables P, Q,

$$\operatorname{div} P = u_t, \qquad Q = \nabla u - P,$$

This research has been done with partial support of KBN, grant number 2 P301 017 05. I would like to thank my colleagues Nick Firoozye, Stefan Müller and Vladimir Šverak for stimulating discussions. Last but not least, I would like to thank Universität Bonn for hospitality; part of this research has been done there.