

RECESSION METHODS IN MONOTONE VARIATIONAL HEMIVARIATIONAL INEQUALITIES

SAMIR ADLY — DANIEL GOELEN — MICHEL THÉRA

Dedicated to Professor Ky Fan on his eightieth birthday

1. Introduction, notations and definitions

Throughout the paper we use standard notations except special symbols introduced when they are defined. All spaces considered are Banach spaces whose norms are always denoted by $\|\cdot\|$. For any space V we consider its dual space V^* equipped with the strong topology. We denote by $\langle \cdot, \cdot \rangle$ the duality pairing between V and V^* . Let $f : V \rightarrow \mathbb{R} \cup \{\infty\}$ be an *extended-real-valued function*. Identifying extended-real-valued functions with their epigraphs

$$\text{epi } f = \{(x, \alpha) \mid x \in V, \alpha \in \mathbb{R} \text{ and } \alpha \geq f(x)\}$$

is a standard tool in convex analysis and in one-sided optimization theory. Also, those functions with closed epigraphs are precisely the lower semicontinuous functions on V , and as usual,

$$\text{dom } f := \{x \in V \mid f(x) < \infty\}.$$

We say that f is *proper* if $\text{dom } f$ is nonempty. In this case $\limsup f(x)$ and $\liminf f(x)$ denote the *upper* and *lower limits* of such (scalar) functions in the classical sense. Depending on context, the symbols $x \xrightarrow{s} y$ and $x \rightharpoonup y$ mean,

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