A SURVEY ON RENORMING AND SET CONVERGENCE

Jonathan M. Borwein¹ — Jon D. Vanderwerff

Dedicated to Ky Fan

Introduction

This survey is intended to complement the authoritative book of Beer [6] as well as his survey paper on Wijsman and other forms of set convergence [9]. These works provide ample historical references as well as applications for various forms of set convergence. With this in mind, let us briefly outline our objectives in this paper.

First, we will present some of the basic properties of dual Kadec–Klee norms that in the context of our study of set convergence are extremely useful. We will often include rather complete arguments because we have yet to find a convenient reference containing the basic facts. The remainder of the first section will discuss some further properties of Kadec–Klee norms. Since *Kadec–Klee* norms, as defined below, play (explicitly or implicitly) an absolutely key role in geometric Banach space theory and its applications, this discussion is significant in its own right. Their utility in the context of set convergence will be abundantly illustrated in our discussion.

After presenting some basic properties of Kadec–Klee norms, we will discuss their interplay with set convergence. The second section gives a brief account of how they entered into the study of set convergence.

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