Topological **M**ethods in **N**onlinear **A**nalysis
Journal of the Juliusz Schauder Center
Volume 6, 1995, 127–135

AN ANALYTIC COMPUTATION OF $ko_{4\nu-1}(BQ_8)$

Boris Botvinnik — Peter Gilkey

Dedicated to Louis Nirenberg

The connective K-theory groups $ko_*(B\pi)$ of a group π appear in many contexts; for example, they are the building blocks for equivariant spin bordism at the prime 2. They also play an important role in the Gromov–Lawson–Rosenberg conjecture which was the starting point of our original investigation [5].

The second author first studied the eta invariant, which is an analytic invariant, whilst a graduate student under the direction of L. Nirenberg so this is perhaps a fitting subject for this volume. In this paper, we will use the eta invariant to determine the additive structure of $ko_{4\nu-1}(BQ_8)$, where

$$Q_8 = \{\pm 1, \pm i, \pm j, \pm k\}$$

is the quaternion group of order 8. We refer to D. Bayen and R. Bruner [2] for an independent topological computation of these groups.

Theorem 1.

(a) $ko_{8\mu+3}(BQ_8) \cong (\mathbb{Z}/2^{3+4\mu}) \oplus (\mathbb{Z}/2^{2\mu}) \oplus (\mathbb{Z}/2^{2\mu+2}) \oplus (\mathbb{Z}/2^{2\mu+2})$.
(b) $ko_{8\mu+7}(BQ_8) \cong (\mathbb{Z}/2^{6+4\mu}) \oplus (\mathbb{Z}/2^{2\mu}) \oplus (\mathbb{Z}/2^{2\mu+2}) \oplus (\mathbb{Z}/2^{2\mu+2})$.

Remark. In fact, we not only determine the additive structure of these groups, our method can also be used to find explicit geometrical generators.

1991 *Mathematics Subject Classification.* Primary 19L41, 58G12; Secondary 55R35, 58G25, 53A50, 55N22.

Key words and phrases. Eta invariant, connective K-theory.

Research of the second author partially supported by the NSF (USA) and IHES (France).