

## DEGREE FORMULAS FOR MAPS WITH NONINTEGRABLE JACOBIAN

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*Dedicated to Professor Louis Nirenberg*

### 1. Introduction

This paper arose from a discussion sparked between the authors after the lecture of Louis Nirenberg at the Conference in Naples on June 1, 1995. He presented a joint work with Haïm Brezis [BN] on the degree theory for VMO (vanishing mean oscillation) mappings  $f : X \rightarrow Y$  between  $n$ -dimensional smooth manifolds. Their results include a variety of discontinuous maps. We soon realized that we can contribute to their work by studying some Orlicz–Sobolev classes weaker than  $W^{1,n}(X, Y)$ . Our approach relies on new estimates for the Jacobians [IS], [GIM] and most recent improvements [I] concerning non-linear commutators. Also  $L^p$ -Hodge theory [S], [ISS] plays a crucial role in this paper.

Let us begin with the well known formula for the degree of a  $C^1$ -map  $f : X \rightarrow Y$ :

$$(1.1) \quad \deg(f; X, Y) = \int_X f^\# \omega,$$

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