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HOMOCLINICS FOR AN ALMOST PERIODICALLY FORCED SINGULAR HAMILTONIAN SYSTEM

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Dedicated to Louis Nirenberg on the occasion of his 70th birthday

0. Introduction

Consider the Hamiltonian system

(HS) $$\ddot{q} + a(t)W'(q) = 0,$$

where a and W satisfy

- (a_1) $a(t)$ is a continuous almost periodic function of t with $a(t) \geq a_0 > 0$ for all $t \in \mathbb{R}$.
- (W_1) There is a $\xi \in \mathbb{R}^2 \backslash \{0\}$ such that $W \in C^2(\mathbb{R}^2 \backslash \{\xi\}, \mathbb{R})$.
- (W_2) $\lim_{x \to \xi} W(x) = -\infty$.
- (W_3) There is a neighborhood $\mathcal{N}$ of ξ and $U \in C^1(\mathcal{N} \backslash \{\xi\}, \mathbb{R})$ such that $|U(x)| \to \infty$ as $x \to \xi$ and
$$|U'(x)|^2 \leq -W(x) \qquad \text{for } x \in \mathcal{N} \backslash \{\xi\},$$
- (W_4) $W(x) < W(0) = 0$ if $x \neq 0$ and $W''(0)$ is negative definite.
- (W_5) There is a constant $W_0 < 0$ such that $\overline{\lim}_{x \to \infty} W(x) \leq W_0$.

When a is periodic in t and somewhat weaker conditions than (a_1) and (W_1)–(W_5) are satisfied, it was shown in [17] that (HS) possesses a pair of solutions that are homoclinic to 0 and wind around ξ in a positive and negative sense

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