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REGULARITY FOR VISCOSITY SOLUTIONS OF FULLY NONLINEAR EQUATIONS $F(D^2u) = 0$

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Dedicated to Louis Nirenberg

1. Introduction

In this paper we study Hölder regularity for the first and second derivatives of continuous viscosity solutions of fully nonlinear equations of the form

$$F(D^2u) = 0. \tag{1.1}$$

It is well known that viscosity solutions of (1.1) are $C^{1,\alpha}$ for some $0 < \alpha < 1$, and in the case that the functional F is convex, they are $C^{2,\alpha}$. In this paper we use the Krylov–Safonov Harnack inequality, Jensen's approximate solutions and some basic lemmas of real analysis to give new and simpler proofs of these results.

In (1.1), u is a real function defined in a bounded domain Ω of $\mathbb{R}^n$ and D^2u denotes the Hessian of u. F is a real-valued function defined on the space $\mathcal{S}$ of real $n \times n$ symmetric matrices. We assume that F is a *uniformly elliptic operator*, that is, for any $M \in \mathcal{S}$ and any nonnegative definite symmetric matrix N,

$$\lambda \|N\| \le F(M+N) - F(M) \le \Lambda \|N\|, \tag{1.2}$$

where $\lambda \le \Lambda$ are two positive constants, which are called *ellipticity constants*, and $\|N\|$ denotes the maximum eigenvalue of N.

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