Topological **M**ethods in **N**onlinear **A**nalysis
Journal of the Juliusz Schauder Center
Volume 6, 1995, 1–30

MORSE RELATIONS FOR GEODESICS ON STATIONARY LORENTZIAN MANIFOLDS WITH BOUNDARY

Fabio Giannoni — Antonio Masiello

Dedicated to Louis Nirenberg on the occasion of his 70th birthday

1. Introduction and statement of the results

Morse theory relates the set of critical points of a smooth functional defined on a Hilbert manifold to the topology of the manifold itself. Morse himself gave the first application of his theory to Riemannian geometry (cf. [6, 11, 12]), proving two very nice and famous results. In order to recall them, consider a Riemannian manifold $(\mathcal{M}, \langle \cdot\,, \cdot \rangle_x)$ with Riemannian structure $\langle \cdot\,, \cdot \rangle_x$.

A curve $\gamma : \,]a, b[\, \to \mathcal{M}$ is said to be a *geodesic* if

$$\nabla_s \dot\gamma(s) = 0 \qquad \text{for any } s \in \,]a, b[,$$

where $\dot\gamma$ is the derivative of γ and $\nabla_s \dot\gamma$ is the covariant derivative of $\dot\gamma$ along γ. It is well known that the geodesic curves joining two given points satisfy a variational principle. Indeed, $\gamma : [0, 1] \to \mathcal{M}$ is a geodesic joining x_0 and x_1 (and defined in the interval $[0, 1]$) if and only if γ is a critical point of the *action integral*

$$f(x) = \frac{1}{2} \int_0^1 \langle \dot x, \dot x \rangle_x \, ds$$

1991 *Mathematics Subject Classification.* 58E05, 58E10, 53C50.
Sponsored by M.U.R.S.T., Research funds, 40%-60%.