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ON THE EXISTENCE OF SIGN CHANGING SOLUTIONS FOR SEMILINEAR DIRICHLET PROBLEMS

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Dedicated to Louis Nirenberg on the occasion of his 70th birthday

1. Introduction

We consider the semilinear Dirichlet problem

$$-\Delta u = f(u) \quad \text{in } \Omega, \quad u = 0 \quad \text{on } \partial\Omega, \tag{D}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with Lipschitz boundary and $f : \mathbb{R} \to \mathbb{R}$ is of class $\mathcal{C}^1$ with $f(0) = 0$. Thus $u_0 \equiv 0$ is a trivial solution of (D) and we are interested in finding and studying nontrivial solutions. One way of obtaining these is to compare the behavior of f near the origin and near infinity. We shall always assume that f grows subcritically at infinity so that variational methods can be applied and the associated functional satisfies the Palais–Smale condition.

Suppose $f'(0) < \lambda_1$ where $0 < \lambda_1 < \lambda_2 \leq \lambda_3 \leq \ldots$ are the eigenvalues (counted with multiplicities) of $-\Delta$ on Ω with homogeneous Dirichlet boundary conditions. If f grows superlinearly at infinity then the mountain pass theorem of Ambrosetti and Rabinowitz [AR], [R] together with the maximum principle guarantees the existence of a positive solution u_+ and a negative solution u_- of (D). Using linking or Morse type arguments Wang [Wa] obtained a third nontrivial solution u_1. In this paper we shall refine Wang's result and obtain more information on u_1 and on other solutions whose existence is proved via Morse theory. Let us illustrate this with the following two theorems. More general results will be stated and proved later.

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