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FIXED POINTS OF MULTIVALUED MAPPINGS IN CERTAIN CONVEX METRIC SPACES

Tomoo Shimizu — Wataru Takahashi

1. Introduction

Takahashi [10] introduced a notion of convexity in metric spaces and studied some fixed point theorems for nonexpansive mappings in such spaces. Let X be a metric space and $I = [0,1]$. A mapping $W : X \times X \times I \to X$ is said to be a *convex structure* on X if for each $(x, y, \lambda) \in X \times X \times I$ and $u \in X$,

$$d(u, W(x, y, \lambda)) \leq \lambda d(u, x) + (1 - \lambda) d(u, y).$$

X together with a convex structure W is called a *convex metric* space.

Recently, Shimizu and Takahashi [9] proved the following result:

Let X be a bounded convex metric space and let T be a multivalued nonexpansive mapping of X into itself such that $T(x)$ is a nonempty compact set for each $x \in X$. Then T has the almost fixed point property in X, i.e.,

$$\inf_{x \in X} d(x, Tx) = 0.$$

In 1974, Lim [5] showed a fixed point theorem for multivalued nonexpansive mappings in uniformly convex Banach spaces. After that, Goebel [2] gave a simpler proof of Lim's theorem using the notion of regular sequences. On the other hand, in 1980, Goebel, Sękowski and Stachura [4] studied hyperbolic

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