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PROPERTIES OF MINIMAL INVARIANT SETS FOR NONEXPANSIVE MAPPINGS

Kazimierz Goebel

In 1965 F. E. Browder [3] and D. Göhde [6] proved that each nonempty bounded and convex subset of a uniformly convex Banach space has the fixed point property for nonexpansive mappings. Also in 1965 W. A. Kirk [8] came to the same conclusion for weakly compact convex subsets of any Banach space under additional assumption that the set has the so-called normal structure. This condition is much weaker than uniform convexity of the space under concern. Since then the problem of finding weaker and weaker conditions implying existence of fixed points for nonexpansive mappings has been the subject of study by many authors. The central themes of these investigations can be found in the book by the author and W. A. Kirk [5].

Many proofs and reasonings in this theory are based on the analysis of a "bizarre" object called "the minimal invariant set".

Let C be a nonempty, weakly compact, convex subset of a Banach space X. Suppose the mapping $T : C \to C$ is *nonexpansive*, i.e. such that

$$\|Tx - Ty\| \leq \|x - y\|$$

holds for all $x, y \in C$.

The set C can contain many "smaller" closed, convex (thus weakly compact) subsets D which are also T-*invariant*, $T(D) \subset D$. Using Zorn's Lemma one

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