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MORSE COMPLEX, EVEN FUNCTIONALS AND ASYMPTOTICALLY LINEAR DIFFERENTIAL EQUATIONS WITH RESONANCE AT INFINITY

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Introduction

I. Motivation. Let H be a Hilbert space and $f : H \to \mathbb{R}$ a C^1-functional. To study critical points of f in the framework of the classical approaches (Morse Theory [39], Ljusternik–Schnirelman theory [40], etc.) one needs to assume, in particular, that f satisfies the Palais–Smale condition (in short, PS-condition): any sequence $\{x_n\} \subset H$ with $\{f(x_n)\}$ bounded and $\nabla f(x_n) \to 0$ contains a convergent subsequence. In turn, the PS-condition is closely related to deformation properties of the flows associated with gradient vector fields. At the same time, as is well-known, there are many important variational problems, where the corresponding functionals fail to satisfy the PS-condition in any suitable sense. In addition, these functionals may not satisfy certain other conditions that are necessary for application of the classical methods.

The problem of weakening the PS-condition has attracted a considerable attention for a long time (see, for instance, [16], [20], [21], [49] and references therein). An essential step in this direction was done by C. Conley [21] who

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