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# THE INTRINSIC MOUNTAIN PASS PRINCIPLE

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## 1. Introduction

Recently M. Schechter proposed new ideas in the variational methods usually known under the name "mountain pass". In [9] he proved a quantitative result of mountain pass type giving up the basic geometrical essence of the classical theorem. Let us fix the abstract mountain pass structure (cf. [4]). We are given a functional $f : X \to R$ defined on a complete metric space $X$, a barrier $B \subset X$, a boundary $A \subset X$ and a family of paths $\Gamma \subset 2^X$ satisfying:

(a) $\gamma \supset A$ for every $\gamma \in \Gamma$,
(b) $\gamma \cap B \neq \emptyset$ for every $\gamma \in \Gamma$,
(c) for any $\gamma \in \Gamma$, $t \geq 0$ and any

$$\eta \in \Xi := \{\theta \in C(X\times[0,1], X) : \eta(x,t) = x \text{ for all } (x,t) \in (X\times\{0\})\cup(A\times[0,1])\}$$

we have $\eta(\gamma, t) \in \Gamma$, i.e. $\Gamma$ is stable with respect to $\Xi$.

Remark 1.1. In case needed one can assume that the family of paths $\Gamma$ is stable with respect to a proper subset of the class of deformations $\Xi$.

In the classical mountain pass setting the functional $f$ is $C^1$ and is "high" on the barrier and "low" on the boundary. In [9] this is violated – the barrier is split into two parts: a "high" part where the functional values are greater

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