ON THE EXISTENCE OF HOMOCLINIC ORBITS FOR THE ASYMPTOTICALLY PERIODIC DUFFING EQUATION

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1. Introduction

In this article we are concerned with the asymptotically periodic Duffing equation in \mathbb{R} , that is

(1.1)
$$-\ddot{u} + u = a(t)|u|^{p-1}u,$$

where p > 1 and $a : \mathbb{R} \to \mathbb{R}$ satisfies:

- (a1) $a \in L^{\infty}(\mathbb{R})$, inf a > 0,
- (a2) $a = a_{\infty} + a_0$, with a_{∞} T-periodic and $a_0(t) \to 0$ as $t \to \pm \infty$.

Noting that 0 is a hyperbolic rest point for (1.1), we look for homoclinic orbits to 0, namely non trivial solutions to (1.1) such that $u(t) \to 0$ and $\dot{u}(t) \to 0$ as $t \to \pm \infty$.

The homoclinic problem for equation (1.1), possibly with a more general nonlinearity, as well as the analogous subcritical elliptic problem on \mathbb{R}^n , has been successfully studied with variational methods by several authors, for different kinds of behaviour of the coefficient a.

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