UNIQUENESS OF PERIODIC SOLUTIONS FOR ASYMPTOTICALLY LINEAR DUFFING EQUATIONS WITH STRONG FORCING

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1. Introduction

In this work we investigate equations of the form

(1)
$$x'' + cx' + ax + g(x) = \lambda p(t),$$

where $g: \mathbb{R} \to \mathbb{R}$ is C^1 , satisfies a Lipschitz condition, and

(2)
$$\lim_{|x| \to \infty} \frac{g(x)}{x} = 0,$$

so that we are dealing with an asymptotically linear problem. The forcing term p is T-periodic, and we are interested in T-periodic solutions of (1). We assume that the linear part of the equation is nonresonant, that is $a \neq 0$ and if c = 0 then $a \neq (2\pi m/T)^2$ for all integer m. Our result shows that for generic forcing term p, when the parameter λ , which measures the strength of the forcing, is sufficiently large, (1) has a unique T-periodic solution.

The existence of a solution (for all λ) under the assumptions made above is a well-known application of degree theory or Schauder's fixed point theorem (see [5] for existence results under much more general conditions), and it is the

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