Topological **M**ethods in **N**onlinear **A**nalysis
Journal of the Juliusz Schauder Center
Volume 12, 1998, 245–261

ON SCHRÖDINGER EQUATION WITH PERIODIC POTENTIAL AND CRITICAL SOBOLEV EXPONENT

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1. Introduction

The main purpose of this paper is to establish the existence of a solution of the semilinear Schrödinger equation

$$-\Delta u + V(x)u = K(x)|u|^{2^*-2}u + f(x,u) \quad \text{in } \mathbb{R}^N, \tag{1}$$

involving a critical Sobolev exponent $2^* = 2N/(N-2)$ with $N \geq 4$ and a subcritical nonlinearity $f : \mathbb{R}^N \times \mathbb{R} \to \mathbb{R}$.

Throughout this paper it is assumed that

(A) The coefficients V and K are continuous and 1-periodic functions in each variable x_i, $i = 1, \dots, N$. Moreover, we assume that $K \geq 0$ on $\mathbb{R}^N$.

In this case it is known that the operator $-\Delta + V$ on $L^2(\mathbb{R}^N)$ has a purely continuous spectrum consisting of closed disjoint intervals. In this paper we consider the case:

(B) 0 is in the spectral gap of the operator $-\Delta + V$.

There are many existence results in the case $K \equiv 0$ on $\mathbb{R}^N$ and we refer to the papers [4], [5], [9], [10], [13].

1991 *Mathematics Subject Classification.* 35J65.
Key words and phrases. Schrödinger equation, critical Sobolev exponent.