

## TYPE II REGIONS BETWEEN CURVES OF THE FUČÍK SPECTRUM AND CRITICAL GROUPS

KANISHKA PERERA — MARTIN SCHECHTER

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### 1. Introduction

The Fučík spectrum arises in the study of semilinear elliptic boundary value problems of the form

$$(1.1) \quad \begin{cases} -\Delta u = f(x, u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where  $\Omega$  is a smooth bounded domain in  $\mathbb{R}^n$  and  $f(x, t)$  is a Carathéodory function on  $\bar{\Omega} \times \mathbb{R}$  such that

$$(1.2) \quad \frac{f(x, t)}{t} \rightarrow \begin{cases} a & \text{a.e. as } t \rightarrow -\infty, \\ b & \text{a.e. as } t \rightarrow \infty. \end{cases}$$

When  $|u(x)|$  is large, (1.1) approximates the equation

$$(1.3) \quad -\Delta u = bu^+ - au^-,$$

where  $u^\pm(x) = \max\{\pm u(x), 0\}$ . The set  $\Sigma$  of those points  $(a, b) \in \mathbb{R}^2$  for which (1.3) (together with the zero boundary condition) has nontrivial solutions is called the Fučík spectrum of  $-\Delta$ .

It was shown in Schechter [7] that, if  $0 < \lambda_1 < \lambda_2 < \dots$  are the distinct Dirichlet eigenvalues of  $-\Delta$ , there are decreasing curves  $C_{11}, C_{12}$  (which may

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