TYPE II REGIONS BETWEEN CURVES
OF THE FUČIK SPECTRUM AND CRITICAL GROUPS

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1. Introduction

The Fučík spectrum arises in the study of semilinear elliptic boundary value problems of the form

$$(1.1) \qquad \begin{cases} -\Delta u = f(x,u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where Ω is a smooth bounded domain in $\mathbb{R}^n$ and $f(x,t)$ is a Carathéodory function on $\overline{\Omega} \times \mathbb{R}$ such that

$$(1.2) \qquad \frac{f(x,t)}{t} \to \begin{cases} a & \text{a.e. as } t \to -\infty, \\ b & \text{a.e. as } t \to \infty. \end{cases}$$

When $|u(x)|$ is large, (1.1) approximates the equation

$$(1.3) \qquad -\Delta u = bu^+ - au^-,$$

where $u^\pm(x) = \max\{\pm u(x), 0\}$. The set Σ of those points $(a,b) \in \mathbb{R}^2$ for which (1.3) (together with the zero boundary condition) has nontrivial solutions is called the Fučík spectrum of $-\Delta$.

It was shown in Schechter [7] that, if $0 < \lambda_1 < \lambda_2 < \ldots$ are the distinct Dirichlet eigenvalues of $-\Delta$, there are decreasing curves C_{l1}, C_{l2} (which may