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ON PARABOLIC QUASI-VARIATIONAL INEQUALITIES AND STATE-DEPENDENT SWEEPING PROCESSES

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1. Introduction

In this paper we consider the evolution problems

$$-u'(t) \in N_{C(t,u(t))}(u(t)) \quad \text{a.e. in } [0,T],\ u(0) = u_0 \in C(0,u_0), \tag{1.1}$$

in a Hilbert space H. We assume that

$$C(t,u) \subset H \text{ is nonempty, closed, and convex for } t \in [0,T],\ u \in H. \tag{1.2}$$

In (1.1), $N_{C(t,u)}(x)$ denotes the normal cone to $C(t,u)$ at $x \in C(t,u)$, cf. Section 2 below. We will treat the case of $(t,u) \mapsto C(t,u)$ being Lipschitz continuous w.r. to the Hausdorff distance d_H with constants $L_1, L_2 \geq 0$, i.e., we require

$$\mathrm{d}_H(C(t,u), C(s,v)) \leq L_1|t-s| + L_2|u-v|, \quad t,s \in [0,T],\ u,v \in H. \tag{1.3}$$

Note that a solution of (1.1) in particular has to satisfy $u(t) \in C(t,u(t))$ for $t \in [0,T]$.

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