Topological **M**ethods in **N**onlinear **A**nalysis
Journal of the Juliusz Schauder Center
Volume 12, 1998, 169–178

VARIATIONAL INEQUALITIES AND SURJECTIVITY FOR SET-VALUED MONOTONE MAPPINGS

C. J. Zhang — Y. J. Cho — S. M. Wei

Throughout this paper, Φ and 2^X denote the real field (or the complex field) and the family of all nonempty subsets of a vector space over Φ, respectively. Let E and F be vector spaces over Φ and $\langle \cdot, \cdot \rangle : F \times E \to \Phi$ be a bilinear functional. For each $x_0 \in E$ and $\varepsilon > 0$, let

$$\omega(x_0, \varepsilon) = \{y \in F : |\langle y, x_0 \rangle| < \varepsilon\}.$$

We denote by $\sigma(F, E)$ the topology on F generated by the family $\{\omega(x, \varepsilon) : x \in E,\ \varepsilon > 0\}$ as a subbase for the neighbourhood system at 0.

It is easy to show that, if F possesses the $\sigma(F, E)$-topology, F becomes a locally convex topological vector space. The $\sigma(E, F)$-topology on E is defined analogously. A subset X of E is said to be $\sigma(E, F)$-compact if X is compact related to the $\sigma(E, F)$-topology.

Let X be a nonempty subset of E. A set-valued mapping $T : X \to 2^F$ is said to be monotone relative to the bilinear functional $\langle \cdot, \cdot \rangle : F \times E \to \Phi$ (monotone for short) if, for all $x, y \in X$, $u \in T(x)$ and $w \in T(y)$,

$$\mathrm{Re}\,\langle u - w, x - y \rangle \geq 0.$$

1991 *Mathematics Subject Classification.* 49J35, 49J40.

Key words and phrases. Monotone and maximal monotone mappings, variational inequality and surjectivity.

The second author was supported by the Academic Research Fund of Ministry of Education, Korea, 1997, Project No. BSRI–97–1405.