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ON A THEOREM OF TVERBERG

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1. Introduction

Let Δ^n denote the n-dimensional simplex. Any face of Δ^n is assumed to be closed. The well-known theorem of Radon (see [6]) can be formulated as follows

Theorem (Radon). *For any linear map $f : \Delta^{n+1} \to \mathbb{R}^n$ there exist two disjoint faces σ_1, σ_2 of Δ^{n+1} such that $f(\sigma_1) \cap f(\sigma_2) \neq \emptyset$.*

In 1966 the Radon theorem was generalized by Tverberg in the following way (see [20]):

Theorem (Tverberg). *For any linear map $f : \Delta^N \to \mathbb{R}^n$, where $N = (p-1)(n+1)$, there exist p pairwise disjoint faces $\sigma_1, \ldots, \sigma_p \subset \Delta^N$ such that*

$$\bigcap_{i=1}^{p} f(\sigma_i) \neq \emptyset.$$

There is a natural question whether the linearity condition for f can be replaced by continuity. The first positive answer was given by Bajmóczy and Bárány in [1] for $p = 2$. Next Bárány, Shlosman and Szücs in [3] proved the theorem for p being a prime number. In 1992 Volovikov obtained the positive answer for any number which is a prime power (see [21]). In all papers mentioned above various generalizations of the classical Borsuk–Ulam antipodal theorem

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