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CONLEY INDEX AND PERMANENCE IN DYNAMICAL SYSTEMS

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1. Introduction

The motivation for our problem comes from permanence theory, which plays an important role in mathematical ecology. Roughly speaking, a flow f on $\mathbb{R}^n \times [0,\infty)$ is said to be permanent (or uniformly persistent) whenever $\mathbb{R}^n \times \{0\}$ is a repeller (see [7]). Other closely related terminology includes cooperativity, persistence and ecological stability. For a discussion of how these terms are related, see [1], [9]. The criterion of permanence for biological systems is a condition ensuring the long-term survival of all species. Sufficient conditions for permanence have been given for a wide variety of models. For more details and extensive bibliographies concerning the problem, we refer the reader to [2], [8].

In this paper we show that if $S \subset \mathbb{R}^n \times \{0\}$ is an isolated invariant set with nonzero homological Conley index, then there exists an x in $\mathbb{R}^n \times (0,\infty)$ such that $\omega(x)$ is contained in S. This may be understood as a strong violation of permanence.

We first give a brief account of the Conley index theory.

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