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GLOBAL BIFURCATION OF OSCILLATORY PERIODIC SOLUTIONS OF DELAY DIFFERENTIAL EQUATIONS

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1. Introduction

In this work we prove a global bifurcation result for periodic solutions of a general class of delay differential equations (allowing for state dependent delays). More precisely, assume that $M > 0$ is any real number and we are given a continuous function $f : C([-M, 0]; \mathbb{R}) \to \mathbb{R}$. Consider the singularly perturbed equation

$$\varepsilon \cdot x'(t) = f(x_t), \quad t > 0,$$

or equivalently (with $\lambda = 1/\varepsilon$)

$$(\mathcal{E}_\lambda) \qquad x'(t) = \lambda \cdot f(x_t), \quad t > 0,$$

where $\lambda > 0$ is a parameter. We will be interested in large values of this parameter. Note that, for every function $y : \mathbb{R} \to \mathbb{R}$ the "translates" $y_t : [-M, 0] \to \mathbb{R}$ are defined by $y_t(\theta) \doteq y(t + \theta)$, where $-M \le \theta \le 0$. Equation $(\mathcal{E}_\lambda)$ includes equations with constant delays (e.g. $f(\phi) = g(\phi(-M))$, $\phi \in C([-M, 0]; \mathbb{R})$, for some $g : \mathbb{R} \to \mathbb{R}$) or state dependent delays (e.g. $f(\phi) = g(\phi(-r(\phi(0))))$, for some $r : \mathbb{R} \to [0, \infty)$).

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