

VARIATIONAL THEOREMS OF MIXED TYPE
AND ASYMPTOTICALLY LINEAR
VARIATIONAL INEQUALITIES

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1. Introduction

To introduce the problem we are going to deal with, let us consider a bounded open subset Ω of \mathbb{R}^N , which will be assumed to be connected and with smooth boundary, a function $g : \Omega \times \mathbb{R} \rightarrow \mathbb{R}$ satisfying the Carathéodory's conditions and a measurable function $\psi : \Omega \rightarrow]-\infty, \infty]$.

We also consider the convex set K_ψ defined by

$$K_\psi = \{u \in W_0^{1,2}(\Omega) \mid u \geq \psi \text{ a.e.}\}.$$

We are interested in finding solutions of the variational inequality

$$(P) \begin{cases} \int_{\Omega} (DuD(v-u) - g(x,u)(v-u) + h(v-u)) dx \geq 0 & \text{for all } v \text{ in } K_\psi, \\ u \in K_\psi, \end{cases}$$

where h is a given function in $L^2(\Omega)$. It is well known that, if $\psi \in W^{2,2}(\Omega)$ and g fulfills some suitable growth conditions with respect to s , then the variational

1991 *Mathematics Subject Classification.* 35A15.

Key words and phrases. Variational inequalities, subdifferentials, topological methods, ∇ -theorems.