Topological Methods in Nonlinear Analysis
Journal of the Juliusz Schauder Center
Volume 12, 1998, 91–107

POSITIVE SOLUTIONS OF QUASILINEAR ELLIPTIC EQUATIONS

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1. Introduction

In this paper we are concerned with the existence of positive solutions for a class of quasilinear elliptic equations of the form

$$\begin{cases} -\Delta_p u = \lambda a(x)|u|^{p-2}u + f(x,u,\lambda), \\ \qquad u \in \mathcal{D}_0^{1,p}(\Omega), \end{cases} \tag{1.1}$$

where Ω is an unbounded domain in $\mathbb{R}^N$ with smooth boundary $\partial\Omega$, $\Delta_p u := \operatorname{div}(|\nabla u|^{p-2}\nabla u)$ $(p > 1)$ is the p-Laplacian, $\mathcal{D}_0^{1,p}(\Omega)$ is the completion of $C_0^\infty(\Omega)$ in the norm $\|u\| = \{\int_\Omega |\nabla u|^p\}^{1/p}$, $0 < a(x) \in L^\infty(\Omega) \cap L^1(\Omega)$, $\lambda \geq 0$ is a real parameter and f satisfies some conditions to be given later.

It is not difficult to show that the eigenvalue problem

$$\begin{cases} -\Delta_p u = \lambda a(x)|u|^{p-2}u, \\ \qquad u \in \mathcal{D}_0^{1,p}(\Omega), \end{cases} \tag{1.2}$$

has the least eigenvalue $\lambda_1 > 0$ with a positive eigenfunction e_1 and λ_1 is the only eigenvalue having this property (cf. Proposition 3.1). This gives us a possibility to study the existence of an unbounded branch of positive solutions bifurcating from $(\lambda_1, 0)$. When Ω is bounded, the result is well-known, we refer to the survey article of Amann [2] and the paper of Ambrosetti and Hess [4] for the case $p = 2$,

1991 *Mathematics Subject Classification.* 35G30, 35J70.
Key words and phrases. Positive solutions, p-Laplacian, fixed point index.