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TRAVELLING WAVES FOR AN INFINITE LATTICE: MULTIBUMP TYPE SOLUTIONS

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1. Introduction

We consider a one dimensional infinite lattice of particles with nearest neighbour interaction

$$(1)\qquad \ddot{q}_k(t) = \overline{V}_x(t, q_{k+1}(t) - q_k(t)) - \overline{V}_x(t, q_k(t) - q_{k-1}(t)), \quad k \in \mathbb{Z}.$$

A solitary wave is a solution of (1) of the form

$$q_k(t) = u(k - ct), \quad k \in \mathbb{Z}.$$

Substituting in (1), we obtain the conditions

$$(2)\qquad \begin{aligned} c^2 u''(k - ct) = {} & \overline{V}_x(t, u(k+1-ct) - u(k-ct)) \\ & - \overline{V}_x(t, u(k-ct) - u(k-1-ct)), \quad k \in \mathbb{Z},\ t \in \mathbb{R}. \end{aligned}$$

Assuming further that $\overline{V}(\,\cdot\,, x)$ is $1/c$-periodic for each $x \in \mathbb{R}$, equations (2) reduce to the second order forward-backward differential-difference equation:

$$(3)\qquad c^2 u''(t) = \overline{V}_x(-t/c, u(t+1) - u(t)) - \overline{V}_x(-t/c, u(t) - u(t-1)).$$

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