Topological **M**ethods in **N**onlinear **A**nalysis
Journal of the Juliusz Schauder Center
Volume 12, 1998, 61–78

ON THE NUMBER OF INTERIOR MULTIPEAK SOLUTIONS FOR SINGULARLY PERTURBED NEUMANN PROBLEMS

Shusen Yan

1. Introduction

In this paper, we will estimate the number of the solutions with exactly k interior local maximum points for the following singularly perturbed problem:

$$\begin{cases} -\varepsilon^2\Delta u + u = u^{p-1} & y \text{ in } \Omega, \\ u > 0 & y \text{ in } \Omega, \\ \dfrac{\partial u}{\partial n} = 0 & y \text{ on } \partial\Omega, \end{cases} \tag{1.1}$$

where ε is a small positive number, Ω is a bounded domain in $\mathbb{R}^N$ with C^1-boundary, n is the unit outward normal of $\partial\Omega$ at $y, 1 < p < (N+2)/(N-2)$ if $N \geq 3$ and $1 < p < \infty$ if $N = 2$.

Much work has been done on (1.1) in the past several years. In [17], [18], Ni and Takagi proved that the least energy solution of (1.1) has exactly one local maximum point x_ε which lies in $\partial\Omega$, and x_ε tends to a point x_0 which attains the maximum of the mean curvature function of $\partial\Omega$. Since then, many authors have constructed solutions for (1.1) with their local maximum points lying in the boundary of Ω. See [2], [5], [8], [12], [15], [21], [22]. Recently, Wei [23], Kowalczyk [13], Bates and Fusco [3] considered the existence of solutions for (1.1),

1991 *Mathematics Subject Classification.* 35J65, 35J20.
Key words and phrases. Nonlinear elliptic equation, peak solution.
This research was supported by ARC.
The author would like to thank Prof. K. Lam for helpful conversation.