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SYMMETRY OF POSITIVE SOLUTIONS OF SOME NONLINEAR EQUATIONS

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1. Introduction

In recent years, a lot of interest has been shown in the study of symmetry properties of solutions of nonlinear elliptic equations, reflecting the symmetry of the domain. In a famous paper, Gidas, Ni and Nirenberg [4] showed that if Ω is smooth, convex and symmetric in one direction, say, that of x_1, then any positive classical solution of the problem

$$\begin{cases} -\Delta u = f(u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

where $f : \mathbb{R} \to \mathbb{R}$ is a locally Lipschitz continuous function, must be also symmetric with respect to x_1. The proof of this result is based on the moving plane method and the maximum principle.

In a recent paper, Berestycki and Nirenberg [2] have substantially simplified the moving plane method obtaining, among other results, the symmetry of the positive solutions of (1.1) without assuming any smoothness on Ω.

When the dimension of the space is two, Lions [9] suggested a method of proving the radial symmetry of positive solutions in a ball when f is positive, without assuming anything on the smoothness of f. While previous results were proved using variants of the moving plane method, this result can be proved using

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