EXISTENCE AND MULTIPLICITY
OF SOLUTIONS FOR SUPERQUADRATIC
NONCOOPERATIVE VARIATIONAL ELLIPTIC SYSTEMS

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1. Introduction and statement of the main results

Let us consider the noncooperative elliptic system

$$
\text{(ES)} \quad
\begin{cases}
-\Delta u = \alpha u - \delta v + F_u(u,v) & \text{in } \Omega, \\
\Delta v = -\delta u - \gamma v + F_v(u,v) & \text{in } \Omega, \\
u = v = 0 & \text{on } \partial\Omega,
\end{cases}
$$

where Ω is a bounded open domain in $\mathbb{R}^N$ with smooth boundary, $\alpha \geq 0$, $\delta \geq 0$, $\gamma \geq 0$ are three real parameters and $F \in C^1(\mathbb{R}^2, \mathbb{R})$.

The solutions of (ES) represent the steady state solutions of reaction-diffusion systems which are derivedfrom several applications, such as mathematical biology or chemical reactions (see for instance [18] and [22]). The following examples are, for instance, particular cases of (ES).

$\lambda - \omega$ **systems.** This kind of system has been widely used as a prototype of reaction-diffusion system

$$
(\lambda - \omega) \quad
\begin{cases}
-\Delta u = \lambda(r)u - \omega(r)v & \text{in } \Omega, \\
\Delta v = -\omega(r)u - \lambda(r)v & \text{in } \Omega, \\
u = v = 0 & \text{on } \partial\Omega,
\end{cases}
$$