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THE EFFECT OF THE GRAPH TOPOLOGY ON A SEMILINEAR ELLIPTIC EQUATION WITH CRITICAL EXPONENT

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1. Introduction

The aim of this paper is to study the effect of the topological structure of the graph of the coefficient $Q(y)$ on the number of the positive solutions of the following elliptic problem:

$$\begin{cases} -\Delta u = Q(y)u^{2^*-1} + \varepsilon u & y \text{ in } \Omega, \\ u > 0 & y \text{ in } \Omega, \\ u = 0 & y \text{ on } \partial\Omega, \end{cases} \tag{1.1}$$

where ε is a small nonnegative number, $2^* = 2N/(N-2)$, $N \geq 4$, Ω is a bounded domain in $\mathbb{R}^N$ with a smooth boundary $\partial\Omega$ and $Q(y)$ is a smooth positive function in $\overline{\Omega}$.

Problem (1.1) stems from differential geometry and has attracted a lot of attention. In the case $\varepsilon > 0$, the existence of at least one solution for (1.1) was established by Brézis and Nirenberg [9] in the case $Q = \text{Const.}$ and by Escobar [12] for a continuous function $Q(y)$ satisfying some additional assumptions. In the case $\varepsilon = 0$, it follows from the Pohozaev identity that problem (1.1) has no solution if Ω is star shaped and $\langle DQ(y), y\rangle \leq 0$. Thus we expect that a solution of problem (1.1) will concentrate at some point as $\varepsilon \to 0+$. So it is

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