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ON JULIUSZ SCHAUDER'S PAPER ON LINEAR ELLIPTIC DIFFERENTIAL EQUATIONS

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Dedicated, with admiration, to Jürgen Moser

In what follows by Ω we denote a bounded domain in $\mathbb{R}^n$ with $n \geq 2$, $\partial\Omega \in C_{2,\alpha}$ and $0 < \alpha < 1$. We let

$$Lu(x) = \sum_{i,k=1}^{n} a_{i,k}(x) \cdot u_{x_i x_k}(x) + \sum_{i=1}^{n} a_i(x) \cdot u_{x_i}(x) + a(x) \cdot u(x) = f(x)$$

to be a linear elliptic differential equation of second order with coefficients a_{ik}, a_i, a, $f \in C_{0,\alpha}(\overline{\Omega})$ and $a(x) \leq 0$ for all $x \in \Omega$.

The paper of J. Schauder "Über lineare elliptische Differentialgleichungen zweiter Ordnung" is treating the solvability of Dirichlet's problem for the above equation. The main results of Schauder's work are given in the following theorems.

Theorem 1 (A priori estimate). *Let $\Omega \subset \mathbb{R}^n$, $n \geq 2$, be a bounded domain with $\partial\Omega \in C_{2,\alpha}$, $0 < \alpha < 1$. There exists a constant $k > 0$, which depends on Ω, $\|a_{ik}\|_{0,\alpha,\Omega}$, $\|a_i\|_{0,\alpha,\Omega}$, $\|a\|_{0,\alpha,\Omega}$, so that for all $u \in C_{2,\alpha}(\overline{\Omega})$ is valid the inequality*[1]

$$\|u\|_{2,\alpha,\Omega} \leq k \cdot \{\|Lu\|_{0,\alpha,\Omega} + \|u\|_{2,\alpha,\partial\Omega}\}.$$

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[1] $C_{k,\alpha}(\overline{\Omega})$ is the Banach space, which is defined as the linear space of k times Hölder continuously differentiable functions in $\overline{\Omega}$ under the norm $\|u\|_{k,\alpha,\Omega} = \sup_{x\in\overline{\Omega}} |u(x)| + \ldots + \sum_{|\beta|=k}\{\sup_{x\in\Omega} |D^\beta u(x)| + H^\alpha_\Omega[D^\beta u]\}$.