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SPATIALLY DISCRETE WAVE MAPS ON (1 + 2)-DIMENSIONAL SPACE-TIME

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Dedicated to Professor Jürgen Moser on the occasion of his 70th birthday

1. Introduction

Let N be a smooth, compact manifold without boundary of dimension k. By Nash's embedding theorem we may assume $N \subset \mathbb{R}^n$ isometrically for some n. A wave map $u = (u^1, \ldots, u^n) : \mathbb{R} \times \mathbb{R}^2 \to N \hookrightarrow \mathbb{R}^n$ by definition is a stationary point for the action integral

$$\mathcal{A}(u; Q) = \int_Q \mathcal{L}(u)\, dz, \quad Q \subset \mathbb{R} \times \mathbb{R}^2,$$

with Lagrangian

$$\mathcal{L}(u) = \frac{1}{2}(|\nabla u|^2 - |u_t|^2)$$

with respect to compactly supported variations u_ε satisfying the "target constraint" $u_\varepsilon(\mathbb{R} \times \mathbb{R}^2) \subset N$. Equivalently, a wave map is a solution to the equation

$$\Box u = u_{tt} - \Delta u = A(u)(Du, Du) \perp T_u N, \tag{1}$$

where A is the second fundamental form of N, $T_p N \subset T_p \mathbb{R}^n$ is the tangent space to N at a point $p \in N$, and "$\perp$" means orthogonal with respect to the standard inner product $\langle \cdot\,, \cdot \rangle$ on $\mathbb{R}^n$.

We denote points on Minkowski space as $z = (t, x) = (x^\alpha)_{0 \le \alpha \le 2} \in \mathbb{R} \times \mathbb{R}^2$ and let $Du = (u_t, \nabla u) = (\partial_\alpha u)_{0 \le \alpha \le 2}$ denote the vector of space-time derivatives.

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