Topological **M**ethods in **N**onlinear **A**nalysis
Journal of the Juliusz Schauder Center
Volume 11, 1998, 283–293

AN EIGENVALUE PROBLEM FOR THE SCHRÖDINGER–MAXWELL EQUATIONS

Vieri Benci — Donato Fortunato

Dedicated to Jürgen Moser

1. Introduction

In this paper we study the eigenvalue problem for the Schrödinger operator coupled with the electromagnetic field $\mathbf{E}, \mathbf{H}$. The case in which the electromagnetic field is given has been mainly considered ([1]–[3]).

Here we do not assume that the electromagnetic field is assigned, then we have to study a system of equations whose unknowns are the wave function $\psi = \psi(x,t)$ and the gauge potentials $\mathbf{A} = \mathbf{A}(x,t)$, $\phi = \phi(x,t)$ related to $\mathbf{E}, \mathbf{H}$.

We want to investigate the case in which $\mathbf{A}$ and ϕ do not depend on the time t and

$$\psi(x,t) = u(x)e^{i\omega t}, \quad u \text{ real function and } \omega \text{ a real number}$$

In this situation we can assume $\mathbf{A} = 0$ and we are reduced to study the existence of real numbers ω and real functions u, ϕ satisfying the system

$$-\frac{1}{2}\Delta u - \phi u = \omega u, \quad \Delta \phi = 4\pi u^2 \tag{1}$$

with the boundary and normalizing conditions

$$u(x) = 0, \quad \phi(x) = g \quad \text{on } \partial\Omega, \quad \|u\|_{L^2} = 1. \tag{2}$$

1991 *Mathematics Subject Classification.* 35Q40.

Key words and phrases. Schrödinger–Maxwell equations, variational methods, nonlinear eigenvalue problems.