

ON r -NEIGHBOURLY SUBMANIFOLDS IN \mathbb{R}^N

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(Submitted by A. Granas)

To Jürgen Moser with very best wishes

1. Introduction

Let M be a k -dimensional manifold, and r a natural number.

DEFINITION. A smooth embedding $M \rightarrow \mathbb{R}^N$ is r -neighbourly if for any r points in M there exists an affine hyperplane in \mathbb{R}^N , supporting M and touching it at exactly these r points.

We denote by $\delta(M, r)$ the minimal dimension N of an Euclidean space, such that there exists an r -neighbourly embedding $M \rightarrow \mathbb{R}^N$, and by $\delta(k, r)$ the maximum of numbers $\delta(M, r)$ over all k -dimensional manifolds M .

The problem of estimating the numbers $\delta(k, r)$ for all k and r was posed by M. Perles in the 1970-ies by analogy with similar problems of combinatorics, and was discussed at the Oberwolfach Combinatorics meetings in 1982 and 1986. Nontrivial examples of r -neighbourly manifolds were constructed in [5]; as G. Kalai communicated to me, in their non-published work with A. Wigderson a polynomial upper estimate $\delta(k, r)$ is proved. However, no nontrivial general lower estimates of these numbers are known.

We consider a similar problem concerning a slightly stronger condition.

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