Topological **M**ethods in **N**onlinear **A**nalysis
Journal of the Juliusz Schauder Center
Volume 11, 1998, 249–271

THE SPACE OF LOOPS ON CONFIGURATION SPACES AND THE MAJER–TERRACINI INDEX

Edward Fadell — Sufian Husseini

Dedicated to Jürgen Moser

1. Introduction

The topology of configuration spaces and their free and based loop spaces plays an important role in the study of the existence of periodic solutions of Hamiltonian systems of n-body type (see e.g. [1], [6], [7]). In particular, if $F_n(\mathbb{R}^k)$ is the configuration space of n-particles $u = (u_1, \dots , u_n)$ in $\mathbb{R}^k$, then the free loop space $\Lambda F_n(\mathbb{R}^k)$ is, up to homotopy type, the domain of a functional f whose critical points are solutions of a corresponding n-body problem. Thus, the homotopy-type invariants of $\Lambda F_n(\mathbb{R}^k)$ such as Lusternik–Schnirelmann (LS) category (and its generalizations — commonly called "index theories"), homotopy and homology play an important role in the subject. In some recent work ([15], [16], [17]), P. Majer and S. Terracini introduced an interesting "collision index" in the space $\Lambda F_n(\mathbb{R}^k)$ which allowed contractions of subsets of $\Lambda F_n(\mathbb{R}^k)$ to move through subspaces intermediate to $\Lambda F_n(\mathbb{R}^k)$ and $\Lambda(R^{nk})$, thus allowing a limited number of collisions during the contraction. However, their index is based upon an equivalence relation which results in a generalized notion of collision. Namely, that u_i "collides" with u_j during the deformation h if there is a chain of indices $i_1, \dots , i_s$, such that $u_i = u_{i_1}$, $u_j = u_{i_s}$ and u_{i_r} collides with $u_{i_{r+1}}$, $1 \leq r \leq s$.

1991 *Mathematics Subject Classification.* Primary 55M30, 55P35; Secondary 58F22.
Key words and phrases. Configuration spaces, loop spaces, Pontryagin algebras, collision index, n-body Hamiltonian systems.