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# ON THE EFFECT OF DOMAIN TOPOLOGY IN A SINGULAR PERTURBATION PROBLEM

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## 1. Introduction

In this paper, we study the following nonlinear elliptic equation

$$\begin{cases} \varepsilon^2 \triangle u - u + u^p = 0 & \text{in } \Omega, \\ u > 0 & \text{in } \Omega \text{ and } u = 0 \text{ on } \partial\Omega, \end{cases} \tag{1.1}$$

where $\Omega \subset \mathbb{R}^N$ $(N \geq 2)$ is a smooth bounded domain, $1 < p < (N+2)/(N-2)$ for $N \geq 3$, $1 < p < \infty$ for $N = 2$ and $\varepsilon > 0$ is a positive small parameter. Our interest in (1.1) arises from two aspects. First, (1.1) is a typical singular perturbation problem. Singular perturbation problems have received much attention lately due to their significances in applications such as chemotaxis (see [18] and [19]), population dynamics (see [1], [16]) and chemical reaction theory (see [1]), etc. Secondly, we are interested in the effect of the properties of the domain, such as geometry, topology on the solutions of nonlinear elliptic problems. Problem (1.1) can be a prototype.

Recently, the geometry of the domain on the solutions of (1.1) has been a subject of study. Beginning in [20], Ni and Wei studied the "*least-energy solutions*" of (1.1) and showed that for $\varepsilon$ sufficiently small, the least-energy solution has only one local maximum point $P_\varepsilon$ and $P_\varepsilon$ must lie in the *most centered* part of $\Omega$, namely, $d(P_\varepsilon, \partial\Omega) \to \max_{P \in \Omega} d(P, \partial\Omega)$, where $d(P, \partial\Omega)$ is the distance from

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