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# ASYMPTOTIC BEHAVIOR OF SOLUTIONS OF 1D-BURGERS EQUATION WITH QUASI-PERIODIC FORCING

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*Dedicated to Jürgen Moser*

## 1. Introduction

We study in this paper the asymptotics of solutions of 1D-Burgers equation

$$\frac{\partial u}{\partial t} + \frac{\partial (u^2)}{\partial x} = \nu \frac{\partial^2 u}{\partial x^2} + \beta F'_\alpha(x) \tag{1}$$

as $t \to \infty$ for the case of quasi-periodic forcing, i.e.

$$F_\alpha(x) = \sum\nolimits_{n=(n_1,\dots,n_d)\in\mathbb{Z}^d} f_n \exp\{2ni(n, \alpha + x\omega)\}.$$

Here $\alpha = (\alpha_1, \dots, \alpha_d) \in \mathrm{Tor}^d$, and $(\alpha + x\omega) \in \mathrm{Tor}^d$ is the orbit of the quasi-periodic flow $\{S^x\}$ on $\mathrm{Tor}^d$, i.e. $S^x\alpha = \alpha + x\omega$, $-\infty < x < \infty$. We shall assume that $\omega$ is Diophantine, i.e. $|(\omega, n)| \geq K/|n|^\gamma$ for positive constants $\gamma$, $K$, $|n| = \sum_{i=1}^d |n_i| \neq 0$. The coefficients $f_n$ decay so fast that $\sum |f_n| \cdot |n|^r < \infty$ for some $r > 1$. Then $F_\alpha$, $F'_\alpha$ can be considered as values of continuous functions $F$, $dF/dx$ on $\mathrm{Tor}^d$ along the orbit of $\{S^x\}$.

The case $d = 1$ was considered in [Si1]. It was shown there that any solution $u(x,t)$ for which $u(x,0)$, $\int_0^x u(y,0)\,dy$ are periodic functions of some period $R$

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