

STABLE MAPS OF GENUS ZERO TO FLAG SPACES

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Dedicated to Professor Jürgen Moser

0. Introduction

Topological quantum field theory led recently to a spectacular progress in numerical algebraic geometry. It was shown that generating functions of certain characteristic numbers of modular spaces of stable algebraic curves with labelled points satisfy remarkable differential equations of *KP*-type (E. Witten, M. Kontsevich). In a later series of developments, this was generalized, partly conjecturally, to the spaces of maps of curves into algebraic varieties leading to the Mirror Conjecture and the construction of quantum cohomology.

The key technical notion in the context of algebraic geometry is that of a *stable map* introduced by M. Kontsevich (cf. [8] and [1]) following the earlier work by M. Gromov in symplectic geometry. It provides a natural compactification of spaces of maps, in the same way as stable curves compactify moduli spaces.

We will be working over a ground field. Let W be an algebraic variety.

DEFINITION 0.1. A stable map (to W) is a structure $(C; x_1, \dots, x_n; f)$ consisting of the following data.

- (a) $(C; x_1, \dots, x_n)$ is a connected complete reduced curve with $n \geq 0$ labelled pairwise distinct non-singular points x_i and at most ordinary double singular points.

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