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ON THE PROBLEM OF REALIZATION OF A GIVEN GAUSSIAN CURVATURE FUNCTION

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Dedicated to Jürgen Moser

1. Introduction

The Gaussian curvature of a smooth surface embedded into the Euclidean space is a smooth function on the surface. We investigate below the local realization problem: given a germ of a smooth function of two variables, find a surface whose Gaussian curvature is the given function.

It is well known that any function germ $g(x,y)$ is realizable as the Gaussian curvature of the surface $z = f(x,y)$ if the curvature value at the central point is not vanishing. It is also known that it is realizable (in the same sense), if the curvature is vanishing at the central point, but its differential does not vanish. In this case, the parabolic curve is smooth.

In the case of a singular parabolic curve, the problem is more difficult. We shall see that any parabolic curve singularity occurs for a suitable surface.

Theorem 1. *For any smooth function of two variables vanishing at its critical point of finite multiplicity, there exists a smooth surface in the Euclidean* 3*-space, whose Gaussian curvature coincides with the given function at a neighbourhood of the given point* (*provided that the surface is identified with the plane by a suitable local diffeomorphism, depending on the function*).

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