

## ON ORBITS OF THE SAME TYPE

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### 1. Introduction

It is well-known that a homeomorphism of  $S^1$  onto itself with a dense orbit is topologically conjugate to an irrational rotation (cf. [7], [8], [11]). We study homeomorphisms of  $(S^1)^n$  with a dense orbit under an additional assumption of equicontinuity related to the rational independence.

In this paper, we concentrate on orbits in a discrete dynamical system induced by a homeomorphism of a Hausdorff uniform space whose closure is homeomorphic to  $S^1$ . We consider a discrete dynamical system that exhibits a connected set  $A$  filled by embedded circles such that each circle is topologically conjugate to an irrational rotation. The hypothesis is that all dynamics within the set are Lyapunov stable, that is, if any two points are close to each other they stay close under the iteration. The main result shows that the rotation numbers of two circles are in  $(1, q)$  resonances and if no twisting occurs the rotation number is constant on  $A$ . To illustrate this fact, we give some examples.

Thus the investigation of the possible behaviour of orbits and of its complexity in a discrete dynamical system yields an index theory for  $\mathbb{Z}$ -actions in connection with  $S^1$ -index. To this end, we show that the existence theorem about continuous extensions holds for  $\mathbb{Z}$ -actions. Moreover, an index theory for group actions is important from the viewpoint of applications to differential equations (cf. [1], [2], [6], [12]).

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