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M-PERIODIC PROBLEM OF ORDER $2k$

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1. Introduction

In monograph [2] the Du Bois–Reymond lemma (fundamental lemma) for periodic functions of order 1 is proved. Next, using the variational method, the authors prove an existence theorem for the periodic problem

$$\begin{aligned} &\ddot{u}(t) = \nabla F(t, u(t)), && t \in [0,T] \text{ a.e.},\\ &u(0) = u(T), && \dot{u}(0) = \dot{u}(T), \end{aligned}$$

in the case when a coercivity condition for the average of F is satisfied and the nonlinearity ∇F is bounded by an integrable function.

In our paper we prove a generalization of the fundamental lemma and then, using the variational method, we give sufficient conditions for the existence of a solution to the following M-periodic problem (matrix-periodic problem)

$$\begin{aligned} (1.1)\quad &\frac{d}{dt}\bigg(\dots\bigg(\frac{d}{dt}\bigg(\frac{d}{dt}u^{(k)} - F_{u_{k-1}}(t,u,\dots,u^{(k-1)})\bigg)\\ &\quad + F_{u_{k-2}}(t,u,\dots,u^{(k-1)})\bigg) + \dots + (-1)^{k-1}F_{u_1}(t,u,\dots,u^{(k-1)})\bigg)\\ &\quad + (-1)^k F_{u_0}(t,u,\dots,u^{(k-1)})\bigg) = 0, \quad t \in [0,T] \text{ a.e.,} \end{aligned}$$

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