Topological **M**ethods in **N**onlinear **A**nalysis
Journal of the Juliusz Schauder Center
Volume 11, 1998, 159–168

ON THE SOLVABILITY OF A TWO POINT BOUNDARY VALUE PROBLEM AT RESONANCE II

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1. Introduction

Let $k \geq 1$ be a fixed integer. We consider the boundary value problem

$$u'' + k^2 u + g(x,u) = h(x) \quad \text{in } (0,\pi), \quad u(0) = u(\pi) = 0, \tag{1_k}$$

where $g : (0,\pi)\times\mathbb{R} \to \mathbb{R}$ is a Carathéodory function, that is, $g(x,u)$ is measurable in $x \in (0,\pi)$ for each $u \in \mathbb{R}$ and continuous in $u \in \mathbb{R}$ for a.e. $x \in (0,\pi)$, $h \in L^1(0,\pi)$ is given. We assume throughout this paper that

(H1) For each $r > 0$, there exists $a_r \in L^1(0,\pi)$ such that

$$|g(x,u)| \leq a_r(x) \quad \text{for a.e. } x \in (0,\pi) \text{ and } |u| \leq r.$$

(H2) There exists $\Gamma \in L^1(0,\pi)$ such that

$$\|\Gamma\|_{L^1} \leq 2k \tag{2}$$

and

$$\limsup_{|u|\to\infty} |g(x,u)/u| \leq \Gamma(x) \tag{3}$$

uniformly for a.e. $x \in (0,\pi)$.

1991 *Mathematics Subject Classification.* 34B15, 47H11.
Key words and phrases. Landesman–Lazer condition, L^1 bound, nonlinear alternative.