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ONE-POINT SINGULAR SOLUTIONS TO THE NAVIER–STOKES EQUATIONS

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Dedicated to Olga Ladyzhenskaya

1. Introduction

Stationary or self similar solutions with suitable homogeneity often play a crucial role in the regularity theory of nonlinear problems, which are physically or geometrically interesting. This has been manifested in the regularity theory of harmonic maps and minimal surfaces. The local partial regularity theorem in [CKN] implies that there are no self-similar solutions with small local energy (also see [TX] for generalizations). Making use of some arguments in [NRS], Tsai has ruled out the existence of any self-similar solutions with a finite local energy. Yet it is unclear whether or not solutions of the incompressible Navier–Stokes equation in three space dimensions would develop singularities in finite time. Therefore, it may be still interesting to construct special solutions of the 3-dimensional Navier–Stokes equation.

In this note, we construct a one-parameter family of explicit smooth solutions of the 3-dimensional impressible Navier–Stokes equation on $\mathbb{R}^3 \setminus p$, where p is any given point. These solutions are axisymmetric, homogeneous of degree -1. They are steady solutions to the Navier–Stokes equations and also solve the self-similar

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