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MULTIPLICITY OF BIFURCATION POINTS FOR VARIATIONAL INEQUALITIES VIA CONLEY INDEX

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1. Introduction

The present work deals with nonlinear eigenvalue problems of the following type:

$$(1.1)\qquad \begin{cases} a(u,v-u)+\langle P'(u),v-u\rangle \geq \lambda b(u,v-u) & \forall v\in\mathbb{K},\\ (u,\lambda)\in\mathbb{K}\times\mathbb{R}, \end{cases}$$

where a,b are bilinear symmetric, $P'(0)=P''(0)=0$, and $\mathbb{K}$ is a closed convex set containing 0. In particular, we search for the λ's such that $(0,\lambda)$ accumulates solutions (u_n,λ_n) with $u_n\neq 0$. It is known that such λ's are eigenvalues of the 0-asymptotic problem, namely there exists $u\neq 0$ such that (u,λ) solves

$$(1.2)\qquad \begin{cases} a(u,v-u)\geq \lambda b(u,v-u) & \forall v\in\mathbb{K}_0,\\ (u,\lambda)\in\mathbb{K}_0\times\mathbb{R}, \end{cases}$$

where $\mathbb{K}_0=\overline{\bigcup_{t>0} t\mathbb{K}}$ is a closed convex cone. The typical problem one has to face is twofold: (1) find eigenvalues (which is nontrivial, unless $\mathbb{K}_0$ is a linear space): (2) ensure that some eigenvalues are bifurcation points (which is not always true, as counterexamples show). Much work has been done in this context; see [11], [15]–[18], [21]–[30], [32]–[34] and the references therein for a more complete picture of the situation.

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