Topological **M**ethods in **N**onlinear **A**nalysis
Journal of the Juliusz Schauder Center
Volume 11, 1998, 45–57

ON THE SOLVABILITY OF A RESONANT ELLIPTIC EQUATION WITH ASYMMETRIC NONLINEARITY

Ana Rute Domingos — Miguel Ramos

1. Introduction

Let Ω be a bounded smooth domain in $\mathbb{R}^N$, $N \geq 1$. In this paper we study the existence of the solution for the elliptic equation with Dirichlet boundary condition

$$-\Delta u = \alpha u^+ - \beta u^- + g(x,u), \quad u \in H_0^1(\Omega), \tag{1.1}$$

where α, β are real parameters and $u^+ = \max\{u,0\}$, $u^- = u^+ - u$. Without loss of generality, we assume $\beta \leq \alpha$. In fact, denoting by (λ_i) the increasing sequence of eigenvalues of $(-\Delta, H_0^1(\Omega))$, we study the case where $\lambda_1 < \beta < \alpha$ and $[\beta,\alpha]$ intersects this linear spectrum. Here $g : \Omega \times \mathbb{R} \to \mathbb{R}$ is a Carathéodory function with *subcritical growth* at infinity, namely $|g(x,s)| \leq A(|s|^{p-1}+1)$ with $1 < p < 2N/(N-2)$ if $N \geq 2$. If $N = 1$, we merely suppose that $|g(x,s)| \leq a(x) + b(x)f(s)$ where $a, b \in L^1(\Omega)$, f is continuous and $f(s) = \mathrm{O}(s)$ near 0.

We consider nonlinear terms which are sublinear at infinity, in a sense to be made precise below (see (2.1)). It is well-known that then the existence and multiplicity of solutions of (D) strongly rely on the position of the pair

1991 *Mathematics Subject Classification.* 35J25, 35J20, 58E05, 34B15.
Key words and phrases. Elliptic equations, variational methods, abstract critical point theory.
Work supported by FCT, PRAXIS XXI, FEDER, project PRAXIS/2/2.1/MAT/125/94 and an EC grant CHRX-CT-94-0555.