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TRAVELLING WAVES FOR REACTION-DIFFUSION-CONVECTION SYSTEMS

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1. Introduction

There is a considerable literature (e.g. [6], [14]) on the existence of travelling-wave solutions of reaction-diffusion equations and systems in the form

$$u_t = Au_{xx} + f(u), \quad u \in \mathbb{R}^N,\ x \in \mathbb{R},\ t \in [0, \infty), \tag{1}$$

where A is a real, positive-definite, $N \times N$ matrix and $f : \mathbb{R}^N \to \mathbb{R}^N$ is a continuously differentiable nonlinear function. The vector u may represent, for example, the concentrations of chemicals or the population densities of interacting species, the interactions between components of u being modelled by the *reaction* term $f(u)$ and their *diffusion* by Au_{xx}. Travelling waves are solutions u of (1) in the form

$$u(x,t) = w(x - ct), \tag{2}$$

where $w : \mathbb{R} \to \mathbb{R}^N$ is the profile of the wave which propagates through the one-dimensional spatially homogeneous domain at the (*a priori* unknown) constant velocity c.

This paper is concerned with an extension of the theory to systems with nonlinear dependence on the gradient of u, such as arise in applications in which

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