Topological Methods in Nonlinear Analysis Journal of the Juliusz Schauder Center Volume 11, 1998, 1–18

MULTIPLICITY RESULTS OF AN ELLIPTIC EQUATION WITH NON–HOMOGENEOUS BOUNDARY CONDITIONS

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1. Introduction

In this paper we study the nonlinear elliptic problem

(1.1)
$$\begin{cases} \Delta u + |u|^{p-2} \ u = 0 & \text{in } \Omega, \\ u = g & \text{on } \partial \Omega, \end{cases}$$

where Ω is an open smooth bounded subset of \mathbb{R}^N , $N \ge 2$, $g : \partial \Omega \to \mathbb{R}$ is a given continuous function and p > 2 is fixed.

If $g \equiv 0$, it is well known that (1.1) has infinitely many distinct solutions for $2 if <math>N \ge 3$ or p > 2 if N = 2. Such results have been proved by using variational methods also for more general odd nonlinearities at the beginning of 70's (see e.g. [2], [3], [6], [9], [11] and references therein). In all these papers a fundamental role is played by the fact that the energy functional is even in a Banach space, hence it is possible to use a modified version of the classical Ljusternik–Schnirelman theory and the properties of the genus for symmetric sets.

On the contrary, if $g \neq 0$ the more general boundary value problem (1.1) loses its symmetry and the previous recalled arguments do not hold. In fact, it is well

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¹⁹⁹¹ Mathematics Subject Classification. 35J20, 35J65.

 $Key\ words\ and\ phrases.$ Non-homogeneous boundary data, non-symmetric variational principle, perturbative methods.

Supported by M.U.R.S.T. (research funds 40 % and 60%) and E.E.C., Program Human Capital Mobility (Contract ERBCHRXCT 940494).