Topological **M**ethods in **N**onlinear **A**nalysis
Journal of the Juliusz Schauder Center
Volume 10, 1997, 387–397

SOLUTIONS OF SUPERLINEAR AT ZERO ELLIPTIC EQUATIONS VIA MORSE THEORY

Vitaly Moroz

Dedicated to the memory of Professor Mark Aleksandrovich Krasnosel'skiĭ

In this note we study the existence of nontrivial solutions of the Dirichlet problem

$$\begin{cases} -\Delta u = f(u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^N$ is an open bounded domain with smooth boundary. We assume that $f \in C(\mathbb{R},\mathbb{R})$ satisfies $f(0) = 0$, so the constant function $u \equiv 0$ is a trivial solution of (1). We are interested in the existence of nontrivial solutions when f is superlinear at zero, that is near zero it looks like $O(u|u|^{\nu-2})$ for some $\nu \in (1,2)$. More precisely, we assume that f and its primitive

$$F(u) = \int_0^u f(\xi)\, d\xi,$$

satisfy the following conditions:

(f_1) for some $\nu \in (1,2)$ there are constants $r, a_r > 0$ such that

$$F(u) \geq a_r |u|^\nu \quad \text{for} \quad |u| \leq r,$$

(f_2) $F(u) - uf(u)/2 > 0$ for all $u \neq 0$.

1991 *Mathematics Subject Classification.* 35J20, 35J60, 58E05.
Key words and phrases. Semilinear elliptic equation, superlinear at zero nonlinearity, multiple solutions, Morse theory.
Supported by the Belorussian Fund of Fundamental Investigations.